OXFORD CAMBRIDGE AND RSA EXAMINATIONS

A LEVEL

H640/01

MATHEMATICS B (MEI)

Pure Mathematics and Mechanics

QUESTION PAPER

WEDNESDAY 6 JUNE 2018: Morning

TIME ALLOWED: 2 hours
plus your additional time allowance

MODIFIED ENLARGED 24pt

YOU MUST HAVE:
Printed Answer Booklet or any suitable paper provided by the centre. The Printed Answer Booklet may be enlarged by the centre.

YOU MAY USE:
a scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

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INSTRUCTIONS

Use black ink. HB pencil may be used for graphs and diagrams only.

Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number or write them on the paper provided.

Answer ALL the questions.

If you use the Printed Answer Booklet WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

The total number of marks for this paper is 100.

The marks for each question are shown in brackets [ ].

You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r|<1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where $${}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x|<1,\ n \in \mathbb{R})$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient Rule $y = \dfrac{u}{v}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h\to 0}\frac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln\left|\mathrm{f}(x)\right| + c$$

$$\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ OR $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \sum(x_i - \bar{x})^2 = \sum x_i^2 - \dfrac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

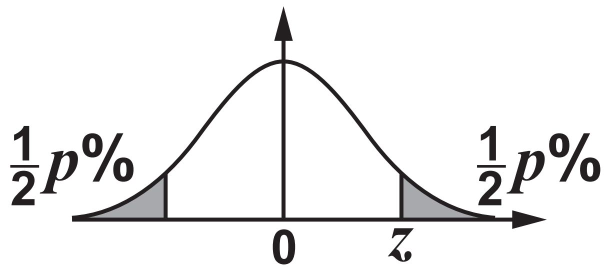
If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r\, p^r q^{n-r}$ where $q = 1 - p$
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Answer ALL the questions

SECTION A (23 marks)

1 Show that $(x-2)$ is a factor of $3x^3-8x^2+3x+2$. [3]

2 By considering a change of sign, show that the equation $e^x-5x^3=0$ has a root between 0 and 1. [2]

3 In this question you must show detailed reasoning.

Solve the equation $\sec^2\theta+2\tan\theta=4$ for $0° \leqslant \theta < 360°$. [4]

4 Rory pushes a box of mass 2.8 kg across a rough horizontal floor against a resistance of 19 N. Rory applies a constant horizontal force. The box accelerates from rest to $1.2\,\mathrm{m\,s^{-1}}$ as it travels 1.8 m.

(i) Calculate the acceleration of the box. [2]

(ii) Find the magnitude of the force that Rory applies. [2]

5 The position vector $\mathbf{r}$ metres of a particle at time t seconds is given by

$$\mathbf{r}=(1+12t-2t^2)\mathbf{i}+(t^2-6t)\mathbf{j}.$$

(i) Find an expression for the velocity of the particle at time t. [2]

(ii) Determine whether the particle is ever stationary. [2]

6 Aleela and Baraka are saving to buy a car. Aleela saves £50 in the first month. She increases the amount she saves by £20 each month.

(i) Calculate how much she saves in two years. [2]

Baraka also saves £50 in the first month. The amount he saves each month is 12% more than the amount he saved in the previous month.

(ii) Explain why the amounts Baraka saves each month form a geometric sequence. [1]

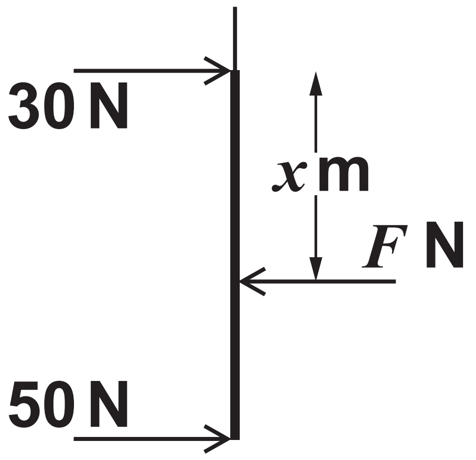
(iii) Determine whether Baraka saves more in two years than Aleela. [3]

Answer ALL the questions

SECTION B (77 marks)

7 A rod of length 2 m hangs vertically in equilibrium. Parallel horizontal forces of 30 N and 50 N are applied to the top and bottom and the rod is held in place by a horizontal force F N applied x m below the top of the rod as shown in Fig. 7.

Fig. 7



(i) Find the value of F. [1]

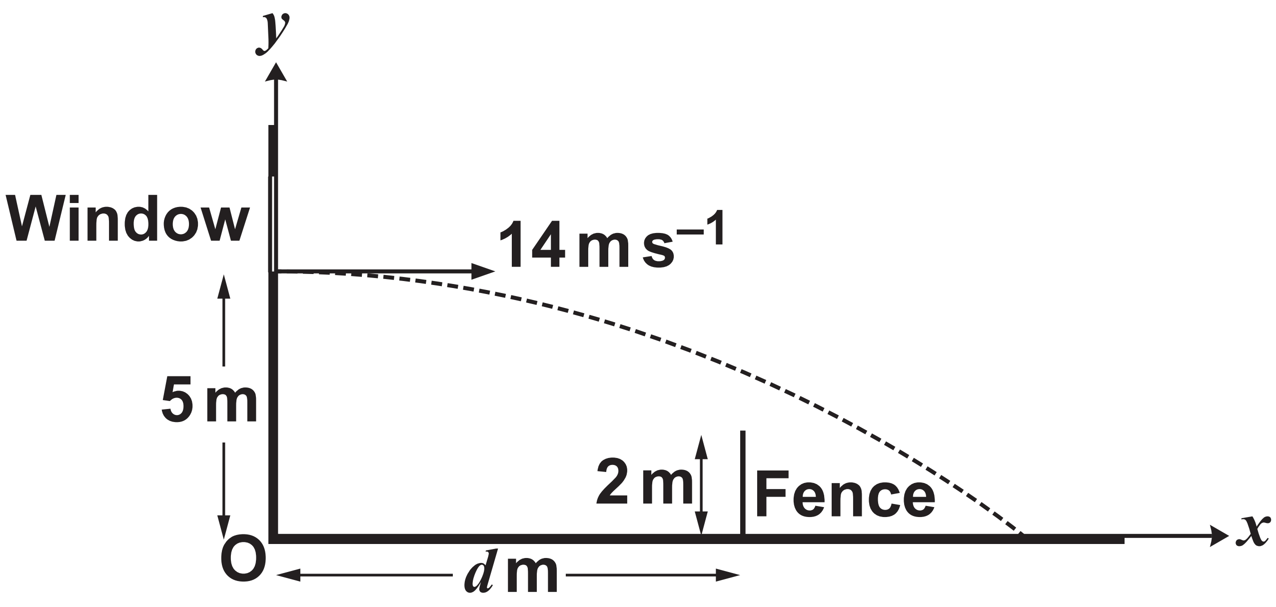
(ii) Find the value of x. [2]

8 (i) Show that $8\sin^2 x\cos^2 x$ can be written as $1 - \cos 4x$. [3]

(ii) Hence find $\int \sin^2 x\cos^2 x\,\mathrm{d}x$. [3]

9 A pebble is thrown horizontally at $14\,\text{m}\,\text{s}^{-1}$ from a window which is $5\,\text{m}$ above horizontal ground. The pebble goes over a fence $2\,\text{m}$ high $d\,\text{m}$ away from the window as shown in Fig. 9. The origin is on the ground directly below the window with the x-axis horizontal in the direction in which the pebble is thrown and the y-axis vertically upwards.

Fig. 9



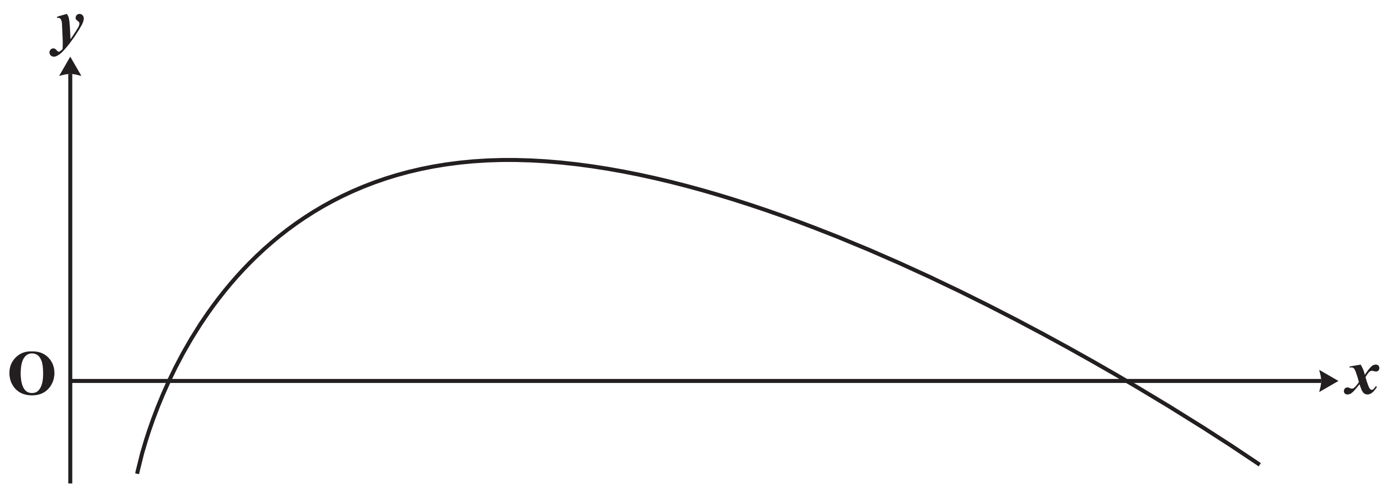
(i) Find the time the pebble takes to reach the ground. [3]

(ii) Find the cartesian equation of the trajectory of the pebble. [4]

(iii) Find the range of possible values for d. [3]

10 Fig. 10 shows the graph of $y = (k - x)\ln x$ where k is a constant ($k > 1$).

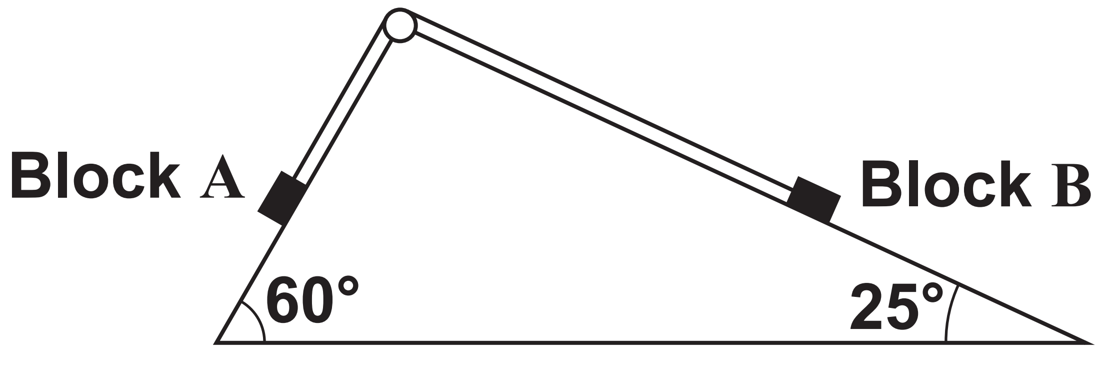
Fig. 10



Find, in terms of k, the area of the finite region between the curve and the x-axis. [8]

11 Fig. 11 shows two blocks at rest, connected by a light inextensible string which passes over a smooth pulley. Block A of mass 4.7 kg rests on a smooth plane inclined at 60° to the horizontal. Block B of mass 4 kg rests on a rough plane inclined at 25° to the horizontal. On either side of the pulley, the string is parallel to a line of greatest slope of the plane. Block B is on the point of sliding up the plane.

Fig. 11

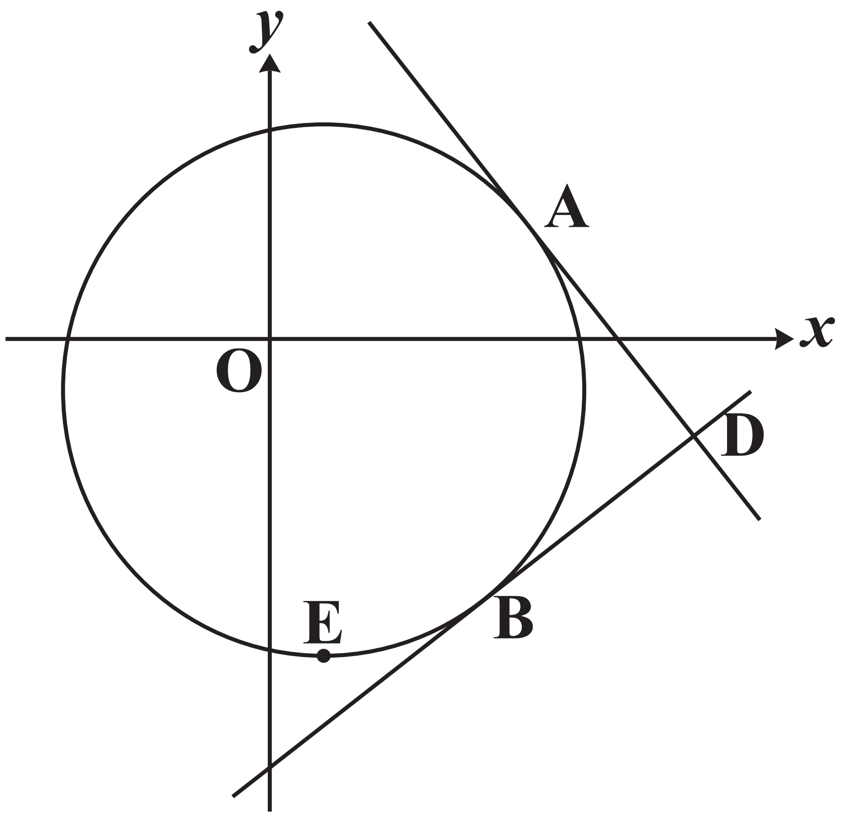


(i) Show that the tension in the string is 39.9 N correct to 3 significant figures. [2]

(ii) Find the coefficient of friction between the rough plane and Block B. [5]

12 Fig. 12 shows the circle $(x-1)^2+(y+1)^2=25$, the line $4y=3x-32$ and the tangent to the circle at the point A (5, 2). D is the point of intersection of the line $4y=3x-32$ and the tangent at A.

Fig. 12



(i) Write down the coordinates of C, the centre of the circle. [1]

(ii) (A) Show that the line $4y=3x-32$ is a tangent to the circle. [4]

(B) Find the coordinates of B, the point where the line $4y=3x-32$ touches the circle. [1]

(iii) Prove that ADBC is a square. [3]

(iv) The point E is the lowest point on the circle. Find the area of the sector ECB. [5]

13 The function $f(x)$ is defined by $f(x) = \sqrt[3]{27 - 8x^3}$. Jenny uses her scientific calculator to create a table of values for $f(x)$ and $f'(x)$.

x	$f(x)$	$f'(x)$
0	3	0
0.25	2.9954	–0.056
0.5	2.9625	–0.228
0.75	2.8694	–0.547
1	2.6684	–1.124
1.25	2.2490	–1.977
1.5	0	ERROR

(i) Use calculus to find an expression for $f'(x)$ and hence explain why the calculator gives an error for $f'(1.5)$. [3]

(ii) Find the first three terms of the binomial expansion of $f(x)$. [3]

(iii) Jenny integrates the first three terms of the binomial expansion of $f(x)$ to estimate the value of

$\int_0^1 \sqrt[3]{27 - 8x^3}\,dx$. Explain why Jenny's method is valid

in this case. (You do not need to evaluate Jenny's approximation.) [2]

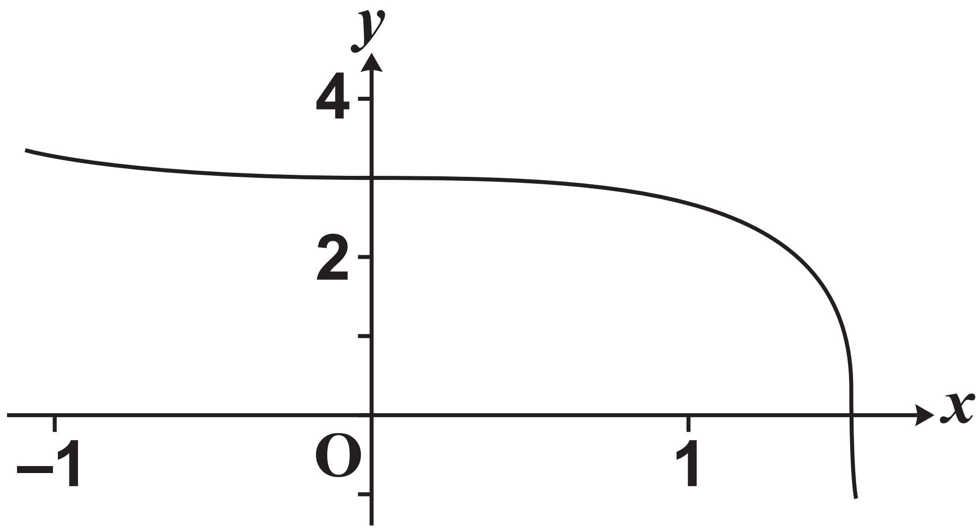
(iv) Use the trapezium rule with 4 strips to obtain an estimate

for $\int_0^1 \sqrt[3]{27 - 8x^3}\,dx$. [3]

The calculator gives 2.921 174 38 for $\int_0^1 \sqrt[3]{27 - 8x^3}\,\mathrm{d}x$.

The graph of $y = \mathrm{f}(x)$ is shown in Fig. 13.

Fig. 13



(v) Explain why the trapezium rule gives an underestimate. **[1]**

14 The velocity of a car, $v\,\mathrm{m\,s^{-1}}$ at time t seconds, is being modelled. Initially the car has velocity $5\,\mathrm{m\,s^{-1}}$ and it accelerates to $11.4\,\mathrm{m\,s^{-1}}$ in 4 seconds.

In model A, the acceleration is assumed to be uniform.

(i) Find an expression for the velocity of the car at time t using this model. [3]

(ii) Explain why this model is not appropriate in the long term. [1]

Model A is refined so that the velocity remains constant once the car reaches $17.8\,\mathrm{m\,s^{-1}}$.

(iii) Sketch a velocity-time graph for the motion of the car, making clear the time at which the acceleration changes. [3]

(iv) Calculate the displacement of the car in the first 20 seconds according to this refined model. [3]

In model B, the velocity of the car is given by

$$v = \begin{cases} 5 + 0.6t^2 - 0.05t^3 & \text{for } 0 \leqslant t \leqslant 8, \\ 17.8 & \text{for } 8 < t \leqslant 20. \end{cases}$$

(v) Show that this model gives an appropriate value for v when $t = 4$. [1]

(vi) Explain why the value of the acceleration immediately before the velocity becomes constant is likely to mean that model B is a better model than model A. [3]

(vii) Show that model B gives the same value as model A for the displacement at time 20 s. [3]

END OF QUESTION PAPER